BPS CALCULATION POLICY

Calculations Policy

The following calculation policy has been devised to meet the requirements of the National Curriculum 2014 for the teaching and learning of mathematics, and is also designed to give pupils a consistent and smooth progression of learning in calculations across the school. Please note that early learning in number and calculation in the Foundation Stage follows the 'Development Matters' EYFS document, and this calculation policy is designed to build on progressively from the content and methods established in the early Years Foundation Stage.

At Beckingham Primary School, we teach for Mastery and follow a Concrete, Pictorial, Abstract approach to the teaching and learning of calculation which we combine with Intelligent Practice and Reasoning throughout. We encourage children to access the fluency of the abstract by visualising and using the concrete and pictorial to the point where fluency is deeply learned. Where a child has the fluency of the number facts. We recognise that they may need the concrete and pictorial to access fluency of problem solving and reasoning and thus ensure that CPA is at the heart of our teaching and learning, is interchangeable and can be used to demonstrate deep learning as well as facilitate it. Our main aim is for children to become fluent and deep thinking mathematicians.

Age Stage Expectations

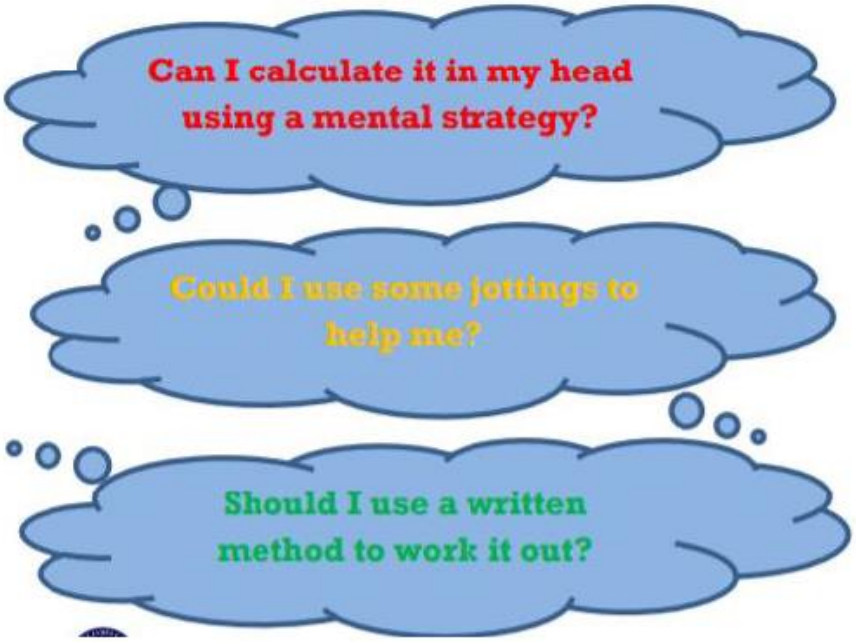
The calculation policy has been organised according to age stage expectations as set out with the National Curriculum 2014, however, pupils' progress in their own learning is fundamental. Therefore, it is vital that pupils are being taught according to the stages that they are currently at, whether this means being moved onto the next stage as soon as they are ready, or working at a lower stage until they are secure enough to move on.

Providing a Context for Calculation

It is important that any type of calculation is given a real life context or problem solving approach to help build the children's understanding of the purpose of calculations and to help them recognise when to use certain operations and methods when faced with problems. This must be a priority within calculation lessons.

Choosing a Calculation Method

Throughout the stages children need to be taught and encouraged to use the following process when deciding what approach they will take to a calculation. This will ensure that they select the most appropriate method for the numbers involved.

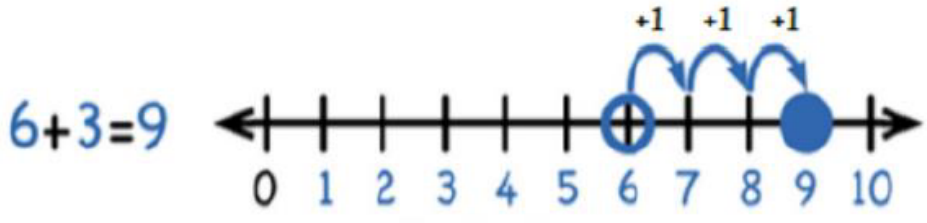


To work out a tricky calculation:
Approximate,
Calculate,
Check it mate!

Addition

Year 1: Add with numbers to 20

Use numbered lines to add by counting on in ones.
Encourage children to start with the larger number and count on.



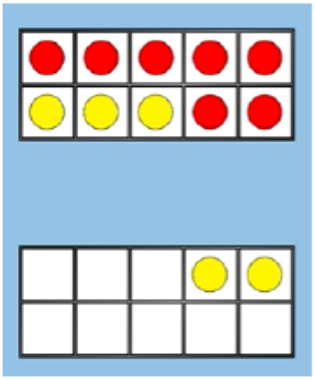
Children should:

- Have access to a wide range of counting equipment, everyday objects, pictorial representations, tens frames and number lines, including dienes and Base 10.
- Be shown number in different contexts
- Read and write the addition (+) and equals to (=) signs within number sentences
- Realise the effect of adding a zero.
- Interpret addition number sentences and solve missing box problems, using concrete objects and number line addition to solve them.

$8 + 3 = \Delta$ $15 + 4 = \Delta$ $5 + 3 + 1 = \Delta$ $\Delta + 3 = 9$ $\Delta + \Delta = 6$

This builds on prior learning of adding by combining two sets of objects into one group (eg 5 cubes and 3 cubes) in EYFS.

Children could use tens frames and double sided counters to show numbers that bridge 10.
Eg 7 + 5 = 12



Key Vocabulary: add, more, plus, and, make, altogether, total, equals to, equals, double, count on, most, number line

Key skills for addition at Year 1:

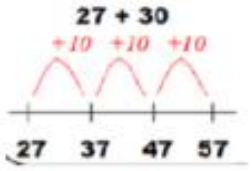
- Count forwards and backwards to and across 100 starting at 0 and 1 or from any given number
- Read and write numbers 1 – 100 in numerals; read and write 1 – 20 in numerals and words
- Read and write mathematical statements including addition (+) and equals to (=)
- Add one digit and two digit numbers to 20 including zero
- Solve one step problems involving addition using concrete objects and pictorial representations including missing number calculations

Addition

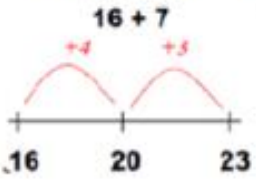
Year 2: Add with 2 digits

Develop mental fluency with addition and place value involving 2 digit numbers, then establish more formal methods

Add 2-digit numbers and tens:



Add 2-digit numbers and ones:

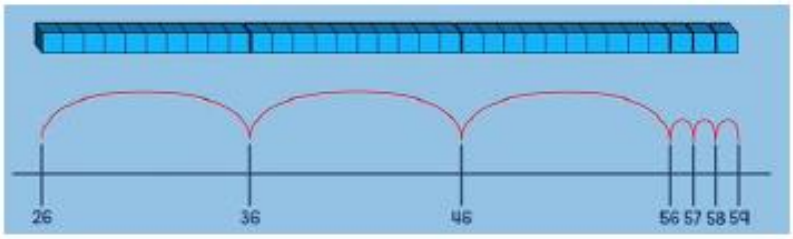


Use empty number lines, concrete equipment, hundred squares etc. to build confidence and fluency in mental addition.

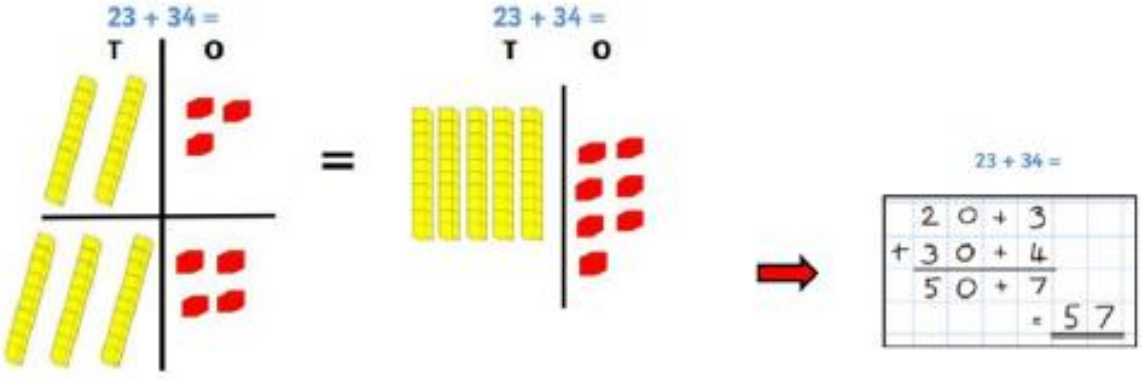
Add pairs of 2 digit numbers, moving to the partitioned column method when secure adding tens and ones.

Step 1: Only provide examples that do **NOT** cross the tens boundary until they are secure with the method itself.

Start with using concrete objects eg Base 10 alongside a number line to show the calculation using jumps that are a relative size:
Eg 26 + 33

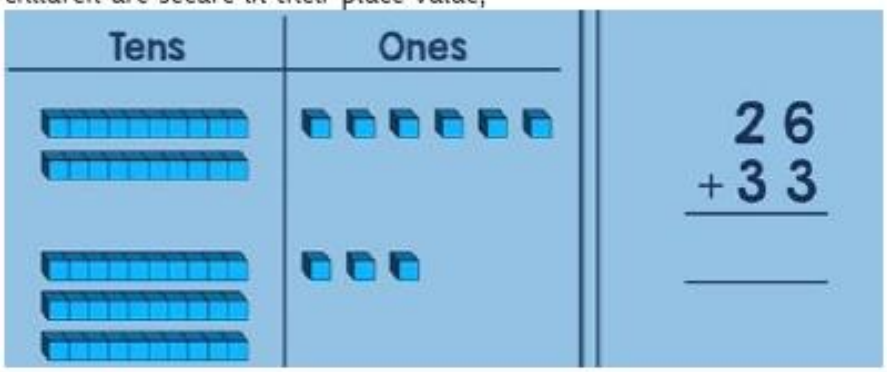


Show the numbers using base 10 and partition them into tens and ones



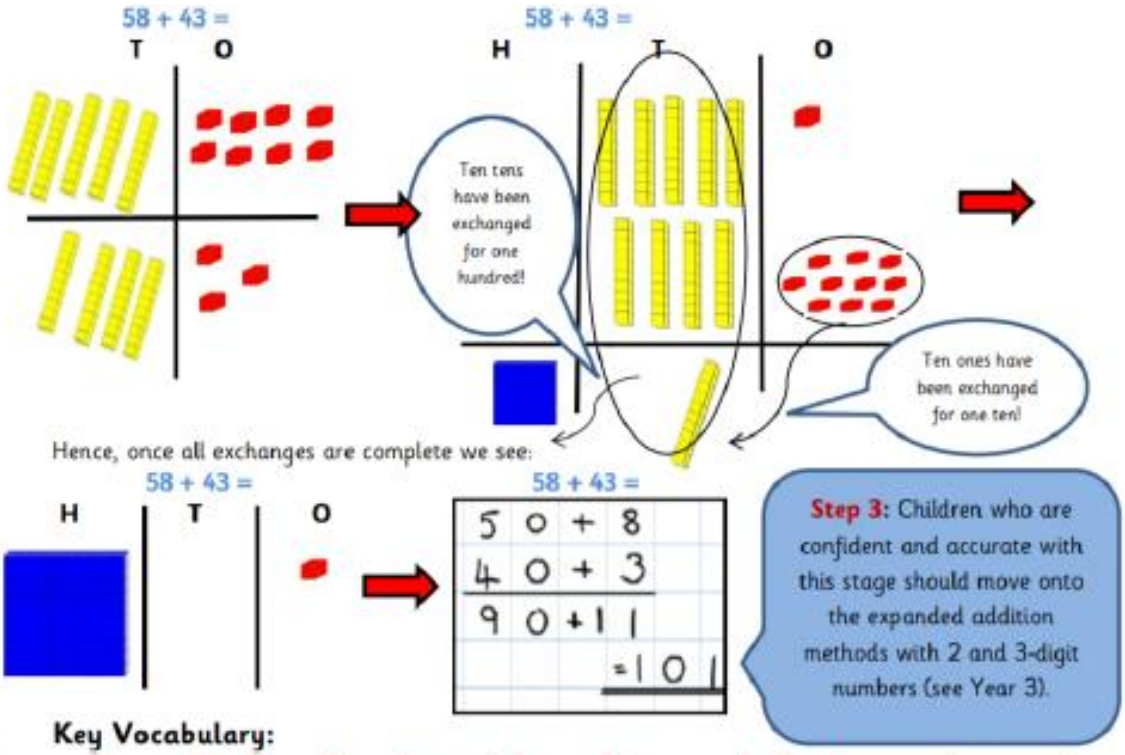
Addition

Then use concrete manipulatives alongside a more formal method still not crossing 10 if children are secure in their place value;



Step 2: Once the children can add a multiple of ten to a 2-digit number mentally (e.g. 80 +11), they are ready for adding pairs of 2-digit numbers that **DO** cross the tens boundary.

To support understanding, pupils may physically carry out the calculation with 'Dienes' apparatus or place value counters, then compare their practical version with the written form, this will help them to build an understanding of it.



Step 3: Children who are confident and accurate with this stage should move onto the expanded addition methods with 2 and 3-digit numbers (see Year 3).

Key Vocabulary:

add, more, plus, and, make, altogether, total, equals to, equals, double, count on, most, number line, sum, tens, ones, partition, addition, columns, tens boundary, hundreds boundary

Addition

Key Skills for addition at Year 2

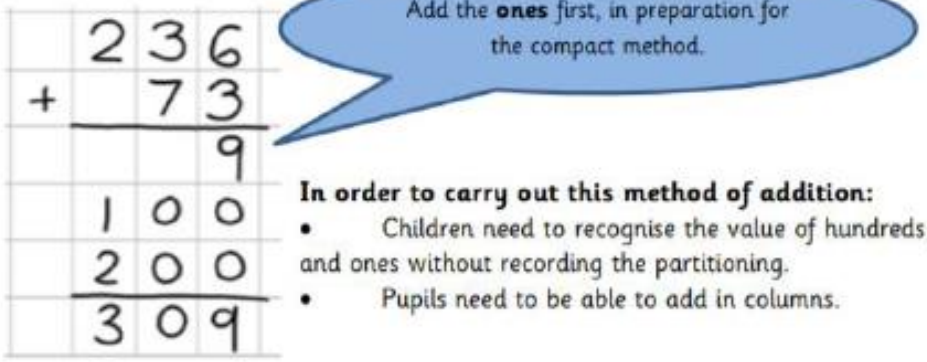
- Count in steps of 2, 3 and 5 from 0, and in tens from any number backwards and forwards
- Read and write numbers to at least 100 in numerals and words
- Identify, represent and estimate numbers using different representations including number lines
- Recognise the place value of each digit in 2 digit numbers
- Compare and order numbers up to 100, using <, >, and = signs
- Recall and use addition facts to 20 fluently
- Show that addition of two numbers can be done in any order (commutative)
- Recognise and use the inverse relationship and use this to check and solve missing number problems
- Add numbers using concrete objects and pictorial representations, and mentally, including:
    - A two digit number and ones
    - A two digit number and tens
    - Two two-digit numbers
    - Adding three one-digit numbers
- Solve addition problems using concrete objects and pictorial representations including those involving numbers, quantities and measures and applying their increasing knowledge of mental and written methods

Addition

Year 3: Add numbers with up to 3 digits

Ensure that the children are secure with the addition methods from Year 2 and then progress onto next steps. Practical apparatus will be required by some children in Year 3 as they progress onto the next stage of addition.

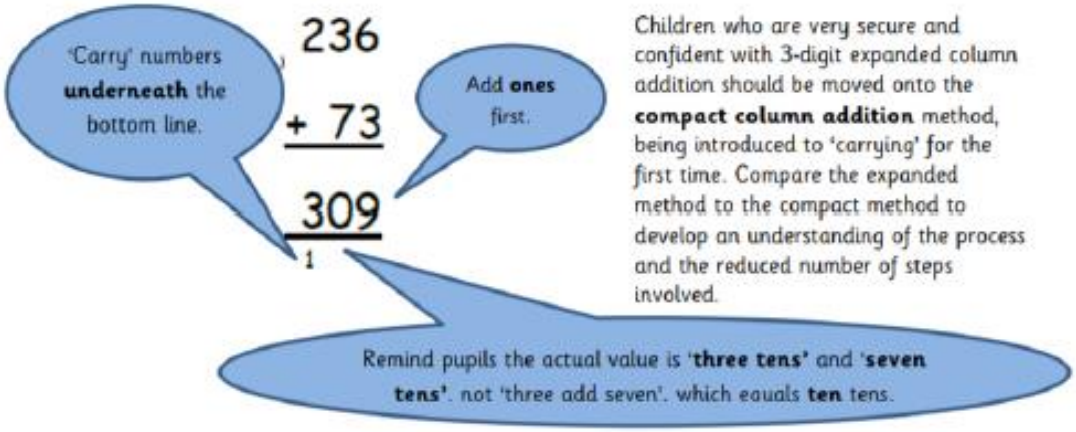
Introduce the **expanded column method.**



In order to carry out this method of addition:

- Children need to recognise the value of hundreds, tens and ones without recording the partitioning.
- Pupils need to be able to add in columns.

Move to the **compact column addition** method, with 'carrying':



Children who are very secure and confident with 3-digit expanded column addition should be moved onto the **compact column addition** method, being introduced to 'carrying' for the first time. Compare the expanded method to the compact method to develop an understanding of the process and the reduced number of steps involved.

Key Vocabulary

add, more, plus, and, make, altogether, total, equals to, equals, double, count on, most, number line, sum, tens, ones, partition, addition, columns, tens boundary, hundreds boundary, hundreds, increase, vertical, carry, expanded, compact

Key Skills for addition at Year 3:

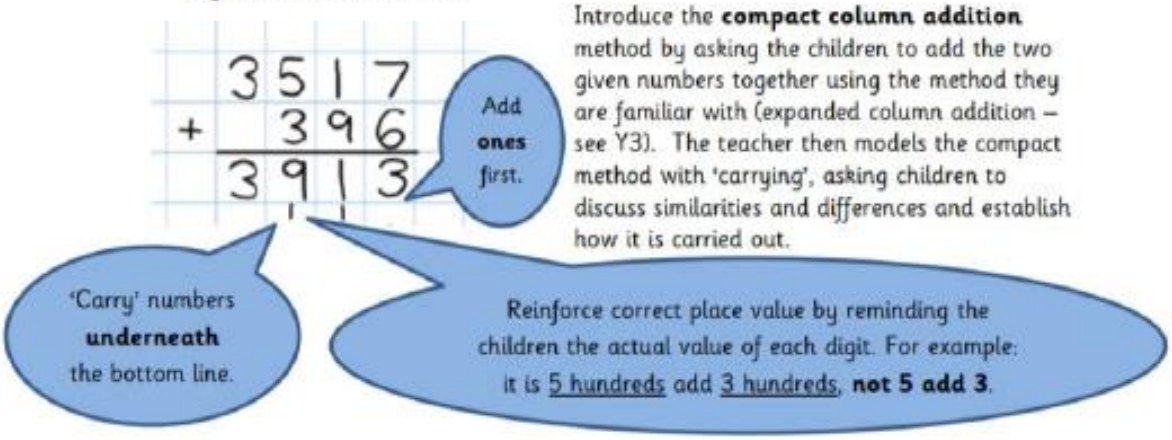
- Read and write numbers to at least 1000 in numerals and words
- Recognise the place value of each digit in a 3 digit number
- Compare and order numbers up to 1000
- Solve number problems and practical problems using these ideas
- Estimate the answer to a calculation and use the inverse operations to check answers
- Add numbers mentally including:
  - A three digit number and ones
  - A three digit number and tens
  - A three digit number and hundreds
- Add numbers up to 3 digits using formal written methods of column addition
- Solve problems including missing number problems, using number facts, place value and more complex addition

Addition

Year 4: Add numbers with up to 4 digits

Move from expanded addition to the compact method, **adding ones first**, and 'carrying' numbers **underneath** the calculation. Also include money and measures contexts.

e.g. 3517 + 396 = 3913



Introduce the **compact column addition** method by asking the children to add the two given numbers together using the method they are familiar with (expanded column addition – see Y3). The teacher then models the compact method with 'carrying', asking children to discuss similarities and differences and establish how it is carried out.

Use and apply this method to money and measures values.

Key Vocabulary

add, more, plus, and, make, altogether, total, equals to, equals, double, count on, most, number line, sum, tens, ones, partition, addition, columns, tens boundary, hundreds boundary, hundreds, increase, vertical, carry, expanded, compact, thousands, digits, inverse

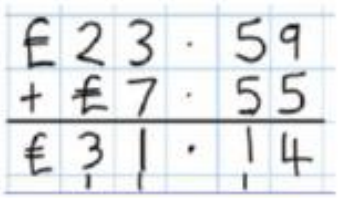
Key skills for addition at Year 4:

- Count in multiples of 6, 7, 9, 25 and 1000
- Recognise the place value of each digit in a 4 digit number
- Order and compare numbers beyond 1000
- Round any number to the nearest 10, 100 or 1000
- Find 1000 more or less than a given number
- Solve number and practical problems with the above and with increasingly large positive numbers
- Estimate and use inverse operations to check answers to a calculation
- Add numbers up to 4 digits using using the formal written method of column addition where appropriate
- Solve addition two-step problems in contexts deciding which operations and methods to use and why
- Continue to practise a wide range of mental addition strategies eg number bonds, using near doubles, partitioning and recombining, adding the nearest 10, 100 or 1000

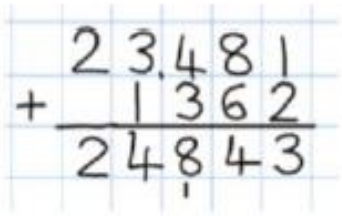
Addition

Year 5: Add numbers with more than 4 digits

This needs to include money, measures and decimals with a differing number of decimal places.
The children should now be fully secure with compact column addition method, understanding the value of each digit and what it represents.



The decimal point should be aligned in the same way as the other place value columns, and must be in the same column in the answer.



Numbers should exceed 4-digits in calculations.

Children should be able to add more than two values, carefully aligning place value columns accordingly.

Say **'6 tenths add 7 tenths'** to reinforce place value.

Empty decimal places may be filled with a zero to highlight the place value in each column.

Children should:

- Understand the place value of **tenths and hundredths,** and use this to align numbers with a different number of decimal places.

Key Vocabulary

add, more, plus, and, make, altogether, total, equals to, equals, double, count on, most, number line, sum, tens, ones, partition, addition, columns, tens boundary, hundreds boundary, hundreds, increase, vertical, carry, expanded, compact, thousands, digits, inverse, decimal places, decimal point, tenths, hundredths, thousandths, integer

Addition

Key Skills for addition at Year 5:

- Read, write, order and compare numbers to at least 1,000,000 and determine the value of each digit
- Round any number up to 1,000,000 to the nearest 10, 100, 1000, 10,000 and 100,000
- Use rounding to check answers to calculations and determine accuracy
- Add whole numbers of more than four digits using column method
- Add numbers mentally with increasingly large numbers
- Solve multi step addition problems in context, deciding which operations and methods to use and why

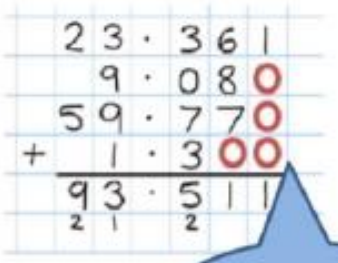
Addition

Year 6: Add several numbers of increasing complexity

Children should be secure and confident to add a variety of numbers in a variety of mathematical contexts

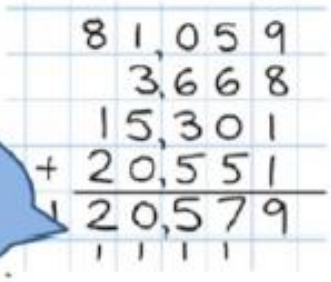
Adding several numbers with a different number of decimal places (including money and measures):

- Tenths, hundredths and thousandths should be correctly aligned, with the decimal point lined up vertically including the answer row.
- Zeros could be added into any empty decimal places, to highlight that there is no value to add.



Empty decimal places may be filled with a zero to highlight the place value in each column.

Adding several numbers with more than 4-digits.



Reinforce correct use of place value and 'Carrying' numbers **underneath** the bottom line.

Key Vocabulary

add, more, plus, and, make, altogether, total, equals to, equals, double, count on, most, number line, sum, tens, ones, partition, addition, columns, tens boundary, hundreds boundary, hundreds, increase, vertical, carry, expanded, compact, thousands, digits, inverse, decimal places, decimal point, tenths, hundredths, thousandths, integer

Key skills in addition at Year 6:

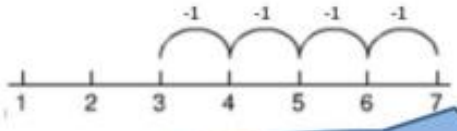
- Read, write, order and compare numbers up to 10,000,000 and determine the value of each digit
- Round any whole number to a required degree of accuracy
- Use negative numbers in context and calculate intervals across zero
- Perform mental calculations including with mixed operations and large numbers
- Use their knowledge of the order of operations to carry out calculations involving all four operations
- Solve addition multi-step problems in context, deciding which operations and methods to use and why
- Children need to utilise and consider a range of mental addition strategies, jottings and written methods before choosing how to calculate

Subtraction

Year 1: Subtract from numbers up to 20

Children consolidate their understanding of subtraction practically, showing subtraction on tens frames, using counters, concrete objects etc. Children work on subtracting in familiar contexts and are introduced to more formal recordings using number lines.

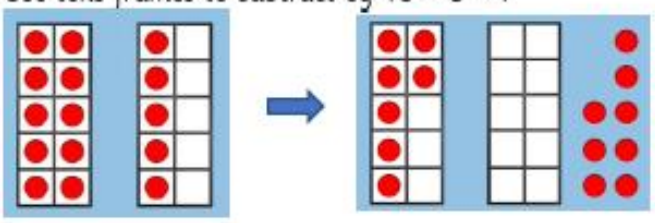
Subtract by taking away:
Count back in ones with a numbered number line to take away with numbers to 20.

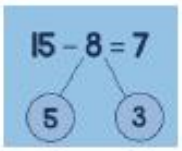


7 – 4 = 3

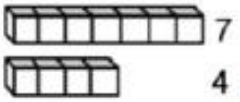
Model subtraction in a variety of ways by using: hundred squares, numbered number lines/tracks and practically.

Use tens frames to subtract eg 15 – 8 = 7





Find the difference between:
This will be introduced practically with the language **'find the difference between'** and **'how many more?'** in a range of familiar contexts.



'Seven is 3 more than four'

I am 3 years older than my sister

Mental subtraction:
Children should start recalling subtraction facts up to and within 10 and 20, and should realise the effect of subtracting zero.

Key Vocabulary

Equal to, take, take away, less, minus, subtract, leave, difference between, how many more, how many less, fewer, most, least, count back, how many left, how much less is _?

Key skills in subtraction at Year 1:

- Given a number identify one more and one less
- Read, write and interpret mathematical statements involving subtraction (-) and (=) sign
- Represent and use number bonds and subtraction facts within 20
- Subtract one digit and two digit numbers to 20 including zero
- Solve one step subtraction problems using concrete objects and pictorial representations and missing number problems
- Read and write numbers to 100 in numerals, and numbers 1 – 20 in numerals and words

Subtraction

Year 2: Subtract with 2 digit numbers

Subtract on a number line by counting back, aiming to develop mental subtraction skills and use dienes equipment to show partitioning in preparation for column subtraction.

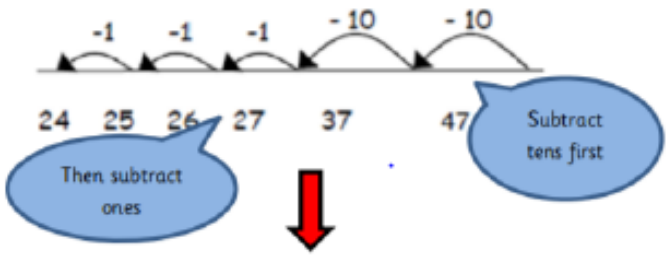
These strategies will be used for:

- 2 digit numbers subtract ones by taking away/ counting back (eg 36 – 7)
- 2 digit numbers subtract tens by taking away/ counting back (eg 64 – 30)
- Subtracting pairs of 2 digit numbers

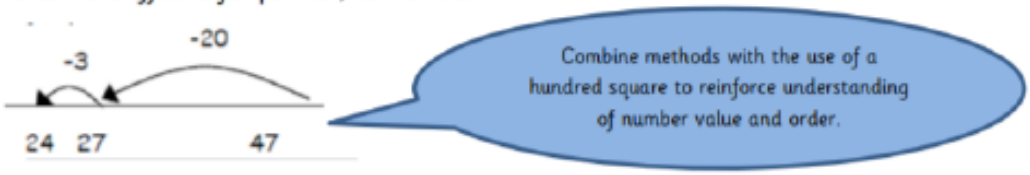
Subtracting pairs of 2-digit numbers on a number line:

47 – 23 = 24

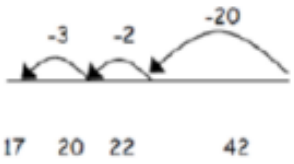
Partition the second number and subtract it in tens and ones, as below:



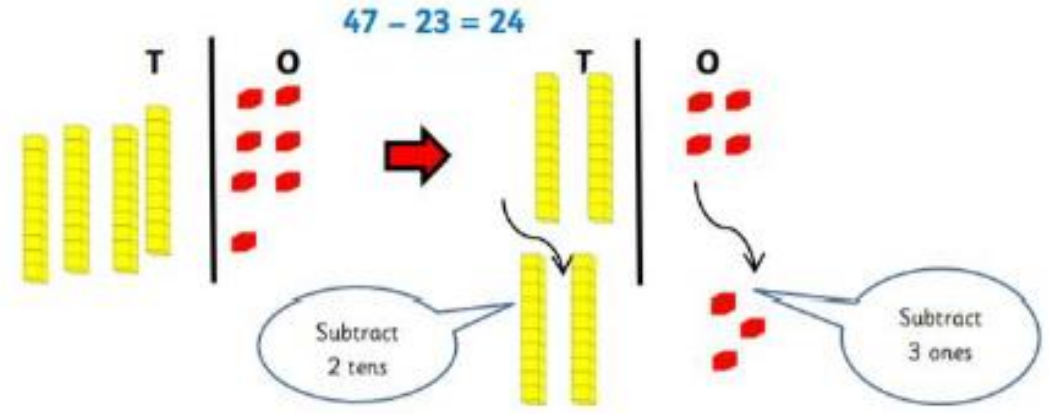
Move towards more efficient jumps back, as seen below:



Teaching children to bridge through ten can help them to become more efficient; for example with 42 – 25 = 17:

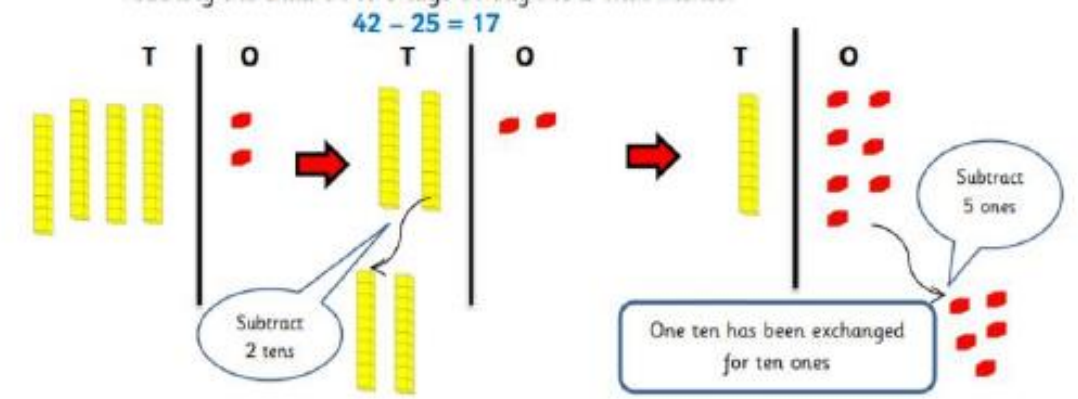


Subtracting pairs of 2-digit numbers using Dienes apparatus:

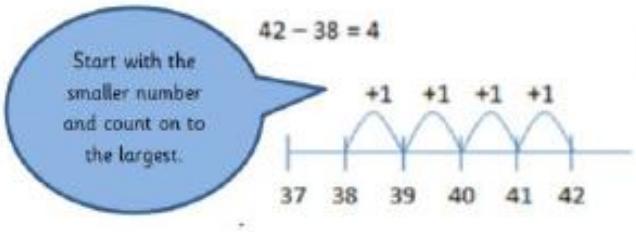


Subtraction

Teaching the children to bridge through tens with Dienes:



Mental strategy – subtract numbers close together by **counting on**:



Children are taught to recognise that when numbers are close together, it is more efficient to count on the difference. They need to be clear about the relationship between addition and subtraction for this.

Key Vocabulary

Equal to, take, take away, less, minus, subtract, leave, difference between, how many more, how many less, fewer, most, least, count back, how many left, how much less is _?, count on, strategy, partition, tens, ones

Key Skills for subtraction at Year 2:

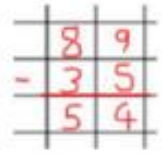
- Read and write numbers to 100 in numerals and words
- Identify, represent and estimate numbers using different representations including number lines
- Recognise the place value of each digit in a 2 digit number
- Compare and order numbers from 0 to 100, using <, > and = signs
- Recall and use subtraction facts to 20 fluently, and derive and use related facts to 100
- Show that subtraction is not commutative
- Recognise and use the inverse relationship to check calculations and solve missing number calculations
- Subtract numbers using concrete objects and pictorial representations and mentally, including:
  - Two digit number and ones
  - Two digit number and tens
  - Two two-digit numbers
- Solve subtraction problems using concrete objects and pictorial representations including numbers, quantities and measures and applying their increasing knowledge of written and mental methods

Subtraction

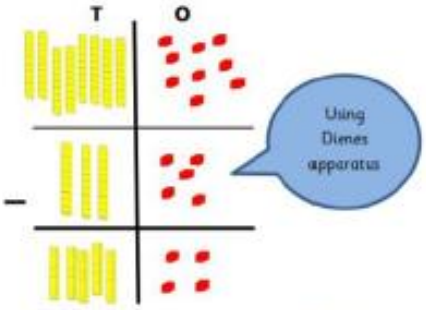
Year 3: Subtracting with 2 and 3 digit numbers

Introduce the children to column method alongside concrete objects and partitioning to ensure understanding

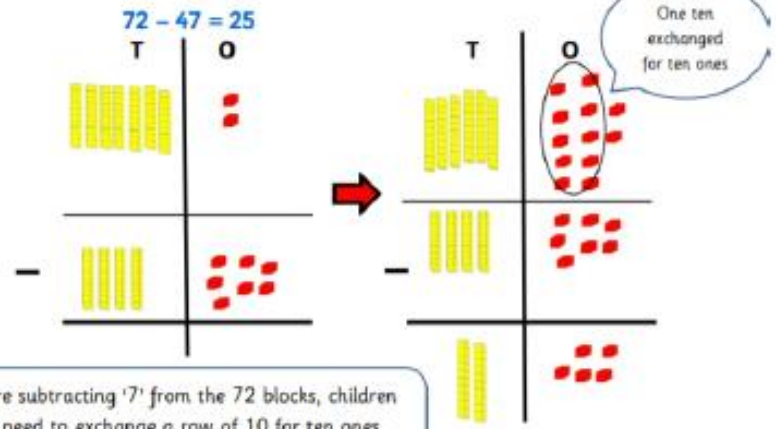
Step 1: Introduce this method with examples where **no exchanging** is required.



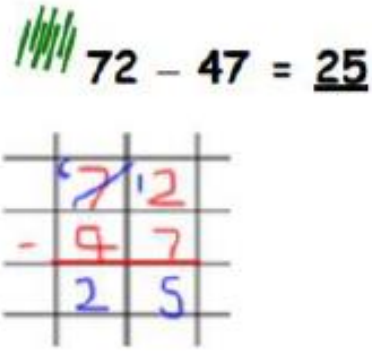
Subtract the ones first (n.b. these used to be known as units), then the tens.



Step 2: Introduce 'exchanging' through practical subtraction. Make the larger number from Dienes apparatus, then subtract 47 from it.



Before subtracting '7' from the 72 blocks, children will need to exchange a row of 10 for ten ones. Then subtract 7 ones, and then subtract 4 tens.

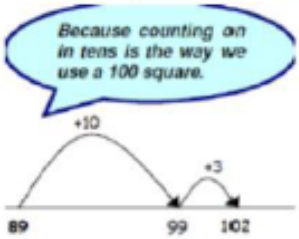


When learning to **'exchange'** in subtraction, explore **'partitioning in different ways'**. Children will then understand that when you exchange, the **VALUE** is the same i.e. 72 = 70 + 2 = 60 + 12 = 50 + 22 etc. Emphasise that the **value hasn't changed**, we have just partitioned it in a different way.

Step 3: Once the children are secure with the understanding of **'exchanging'**, they can use column method to subtract any 2 and 3 digit numbers.

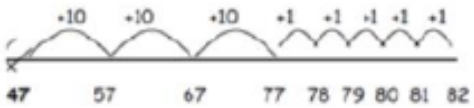
Subtraction

Counting on as a mental strategy for subtraction:



Continue to reinforce counting **on** as a strategy for **close-together numbers** (e.g. 121-118), and also for numbers that are 'nearly' multiples of 10, 100, 1000 or £s, which make it easier to count on (e.g. 102 – 89, 131 – 79, or calculating change from £1 etc)

- Start at the smaller number and count on **in tens first**, then count in in ones to find the rest of the difference: 82 – 47 = 35



Key Vocabulary:

Equal to, take, take away, less, minus, subtract, leave, difference between, how many more, how many less, fewer, most, least, count back, how many left, how much less is _?, count on, strategy, partition, tens, ones, exchange, decrease, hundreds, value, digit

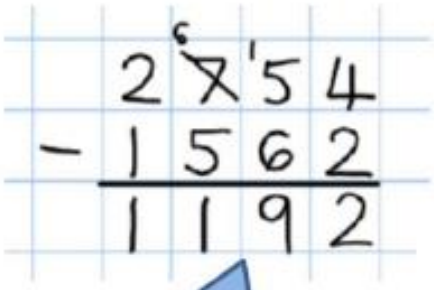
Key Skills for subtraction at Year 3:

- Subtract mentally:
  - 3 digit number and ones
  - 3 digit number and tens
  - 3 digit number and hundreds
- Subtract numbers up to 3 digits using formal written method of column subtraction
- Estimate the answer to a calculation and use inverse operations to check answers
- Solve subtraction problems including missing number problems, using number facts, place value and more complex subtraction
- Find 10 or 100 less than a given number
- Recognise the place value of each digit in a 3 digit number
- Counting up differences as a mental strategy when numbers are close together or near multiples of 10
- Read and write numbers up to 1000 in numerals and words
- Practise mental subtraction strategies, such as subtracting near multiples of ten and adjusting (eg 19 or 21) and select most appropriate methods to subtract explaining why

Subtraction

Year 4: Subtract with up to 4 digit numbers

Compact column subtraction with exchanging (decomposition)



As introduced in year 3 but with more complex numbers.

Use place value counters to reinforce exchanging.

Give plenty of opportunities to apply this method to money and measures

Always encourage children to consider the best method for the numbers involved – mental, counting on, counting back or written method.

Mental strategies

A variety of mental strategies must be taught and practised including counting on to find the difference where numbers are close together or where it is easier to count on.

Key vocabulary:

Equal to, take, take away, less, minus, subtract, leave, difference between, how many more, how many less, fewer, most, least, count back, how many left, how much less is _?, count on, strategy, partition, tens, ones, exchange, decrease, hundreds, value, digit, inverse

Key Skills for subtraction at Year 4:

- Count backwards through zero including negative numbers
- Find 1000 less than a given number
- Recognise the place value in each digit of a four digit number
- Order and compare numbers beyond 1000
- Round any number to the nearest 10, 100 or 1000
- Solve number and practical problems with increasingly large positive numbers
- Estimate and use inverse operations to check answers to calculations
- Subtract numbers with up to 4 digits using formal written method of column subtraction
- Solve two step subtraction problems in context, deciding which operations and methods are best to use and why
- Subtract by counting on where numbers are close together or are near multiples of 10, 100 or 1000

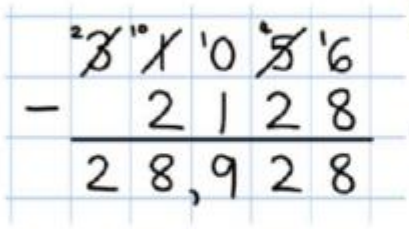
Subtraction

Year 5: Subtract with at least 4 digit numbers

This needs to include money, measures and decimals with a differing number of decimal places.

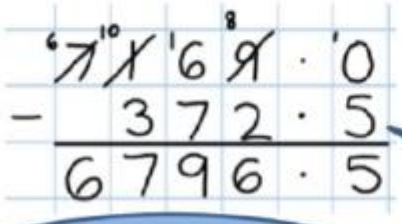
The children should all be fully secure with column subtraction and understand the value of each digit and what it represents.

Compact column subtraction:
(with 'exchanging' – decomposition)



Children who are still not secure with number facts and place value will need to remain on the partitioned column method until they are mathematically ready for the compact method.

Subtracting with larger integers.



Subtract with decimal place values, including mixtures of integers and decimals, aligning the decimal point accurately.

Add a 'zero' in any empty decimal places to aid the understanding of what to subtract in that specific place value column.

Create and provide lots of opportunities for subtracting and finding differences with money and measures.

Key Vocabulary

Equal to, take, take away, less, minus, subtract, leave, difference between, how many more, how many less, fewer, most, least, count back, how many left, how much less is _?, count on, strategy, partition, tens, ones, exchange, decrease, hundreds, value, digit, inverse, integer, tenths, hundredths, decimal, decimal point

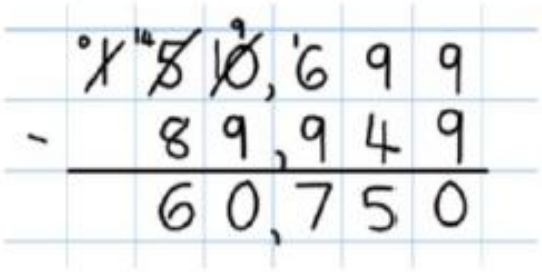
Key Skills for subtraction at Year 5:

- Subtract numbers mentally with increasingly large numbers
- Subtract numbers with more than 4 digits using formal written method of column subtraction
- Solve subtraction multi step problems in context, deciding which operations and methods to use and why
- Use rounding to check answers to calculations and determine levels of accuracy
- Read, write, order and compare numbers to at least 1,000,000 and determine the value of each digit
- Count backwards in steps of powers of ten for any given number up to 1 million
- Interpret negative numbers in context, counting backwards with positive and negative integers through zero
- Round any number up to 1 million to the nearest 10, 100, 1000, 10,000 and 100,000

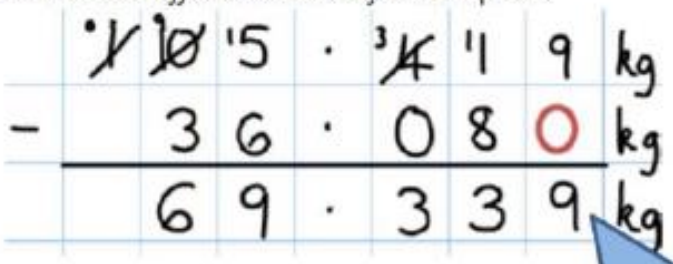
Subtraction

Year 6: Subtracting with increasingly large and more complex numbers and decimal values

Children at this stage should be secure with compact column method for subtraction. They will now be progressing to using this method to subtract more complex integers.



Children will be using the compact column method to subtract money and measures, including decimals with different numbers of decimal places.



Children should be able to apply their knowledge of a range of mental strategies, mental recall skills, and informal and formal written methods when selecting the **most appropriate method** to work out subtraction problems.

Empty decimal places can be filled with 'zero' to show the place value in each column.

Key Vocabulary

Equal to, take, take away, less, minus, subtract, leave, difference between, how many more, how many less, fewer, most, least, count back, how many left, how much less is _?, count on, strategy, partition, tens, ones, exchange, decrease, hundreds, value, digit, inverse, integer, tenths, hundredths, decimal, decimal point

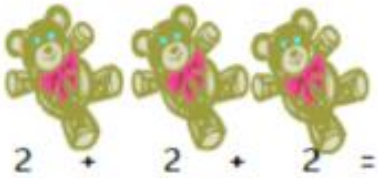
Key Skills for subtraction at Year 6:

- Perform mental calculations, including with mixed operations and large numbers, using and practising a range of mental strategies
- Children understand how to subtract mentally with larger numbers and calculations of increasing complexity
- Solve multi-step problems in context, deciding which operations and methods to use and why
- Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy
- Read, write, order and compare numbers up to 10 million and determine the value of each digit
- Round any whole number to a required degree of accuracy
- Use negative numbers in context and calculate intervals across zero
- Children need to utilise and consider a range of mental subtraction strategies, jottings and written methods before choosing how to calculate

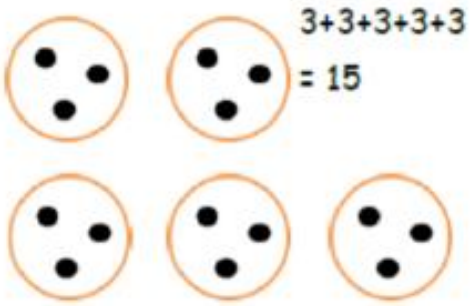
Multiplication

Year 1: Multiply with concrete objects, arrays and pictorial representations

How many legs will 3 teddies have?

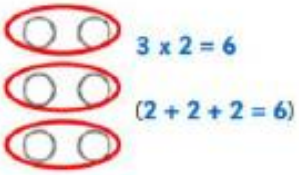


There are 3 sweets in one bag.
How many sweets are in 5 bags altogether?



- Give children experience of counting in equal groups of 2s, 5s and 10s using concrete objects to do this
- Present practical problem solving activities involving counting equal sets or groups as shown above

Use arrays with support of teacher



Key Vocabulary

Groups of, lots of, times, array, altogether, multiply, count

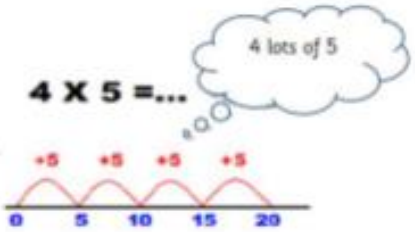
Key Skills for multiplying at Year 1:

- Count in multiples of 2, 5 and 10
- Solve one-step problems involving multiplication, by calculating the answer using concrete objects, pictorial representations and arrays with support of the teacher
- Make connections between arrays, number patterns and counting in 2s, 5s and 10s
- Begin to understand doubling using concrete objects and pictorial representations

Multiplication

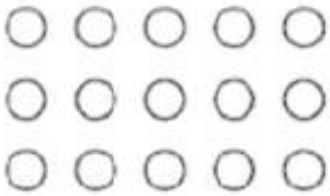
Year 2: Multiply using arrays and repeated addition using at least 2s, 5s and 10s

Use repeated addition on a numberline
Starting from zero, make equal jumps on a numberline to work out multiplication facts and write multiplication statements using x and = signs.



4 x 5 = 20

Use arrays



3 x 5 = 15
3 x 5 = 5 + 5 + 5 = 15

5 x 3 = 15
3 x 5 = 3 + 3 + 3 + 3 + 3 = 15

Use arrays to help teach children to understand the commutative law of multiplication and give examples such as 3 x __ = 6.

Use practical apparatus

3 x 5 = 5 + 5 + 5 = 15

Use mental recall:

- Children need to begin to recall multiplication facts for the 2, 5 and 10 times table through practise in counting and understanding of the operation.
- Children may begin to recall multiplication facts for the 3 times table based on the work of counting in steps of 3 from zero.

Multiplication

Key Vocabulary

Groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative law, sets of, lots of, equal groups, times as big as, once, twice, three times…

Key Skills for multiplication at Year 2

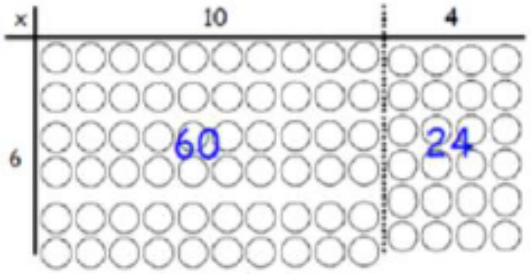
- Count in steps of 2, 3 and 5 from zero, and in tens from any number
- Recognise and use multiplication facts from the 2, 5 and 10 times table, including recognising odds and evens
- Write and calculate statements for multiplication within the multiplication tables using the x and = signs
- Show that multiplication can be done in any order (commutative)
- Solve a range of problems involving multiplication, using concrete objects, arrays, repeated addition, mental methods and multiplication facts including problems in context
- Children use a variety of language to discuss and describe multiplication

Multiplication

Year 3: Multiply 2 digits by a single digit

Introduce informal methods for multiplying 2 digits by a single digit
Children can make sense of this by visualising the multiplication by linking the layout of a grid to that of arrays initially

14 x 6 = 84



60 + 24 = 84

(10 x 6) + (6 x 4)
60 + 24 = 84

Introduce the informal partitioning method with children physically making an array to represent the calculation (eg make 8 lots of 23 with 10s and 1s place value counters) then translate this to partitioned format.

23 x 8 = 184

160 + 24 = 184

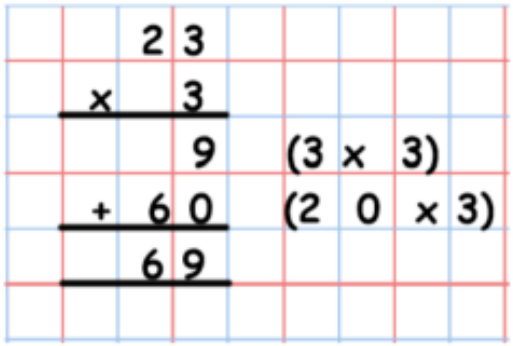
(20 x 8) + (3 x 8)

160 + 24 = 184

Multiplication

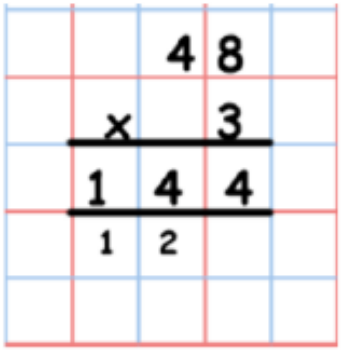
Progress to formal methods for multiplying 2 digit by 1 digit numbers:
Once secure with partitioning and related facts, progress the children onto multiplying 2 digits by 1 digit with regrouping.

23 x 3 = 69



Then progress onto the short method of multiplication for multiplying 2 digits by 1 digit.

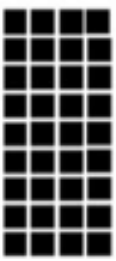
48 x 3 = 144



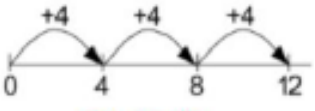
To do this, children must be able to:

- Partition numbers into tens and ones
- Multiply multiples of ten by a single digit (eg 80 x 4 = 320) using their knowledge of place value facts and multiplication
- Recall and calculate multiplication facts for the 2, 3, 4, 5, 8 and ten times table
- Calculate unknown multiplication facts by repeated addition or other taught mental strategies (eg by commutative law, working out near multiples and adjusting, using doubling etc)

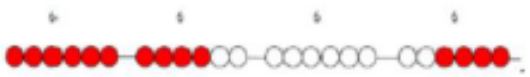
Strategies to support the last bullet point are repeated addition using a number line, arrays and bead strings.



9 x 4 = 36



3 x 4 = 12



4 x 6 = 24

Multiplication

Key Vocabulary

Groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative law, sets of, lots of, equal groups, times as big as, once, twice, three times…, partition, total, multiple, product, tens, ones, value

Key Skills for multiplication at Year 3:

- Count from zero in multiples of 4, 8, 50 and 100
- Recall and use multiplication facts from the 2, 3, 4, 5, 8 and 10 times table and multiply multiples of ten
- Write and calculate number statements using the multiplication tables they know, including 2 digit by single digit, drawing upon mental methods and progressing to reliable written methods
- Solve multiplication problems including missing number problems
- Develop mental strategies using commutativity and associativity (eg 4 x 12 x 5 = 4 x 5 x 12 = 20 x 12 = 240)
- Multiply whole numbers by 10 and 100
- Solve simple problems in context, deciding which operations and methods to use
- Develop efficient mental methods to solve a range of problems, including missing number problems

Multiplication

Year 4: Multiply 2 and 3 digits by a single digit

Children need to be using all multiplication tables up to 12 x 12

Developing the short formal method and progressing in year four:
See 2 digit by 1 digit method in year 3. Progress to 3 digit by 1 digit.

135 x 4 = 540

	1	3	5					
x			4					
		2	0	(5	x	4)		
	1	2	0	(3	0	x	4)	
	4	0	0	(1	0	0	x	4)
	5	4	0					

Progressing to: **136 x 6 = 816**

	1	3	6	
x			6	
	8	1	6	
	2	3		

Pupils could be asked to work out a given calculation using the partitioning method (see Y3), and then compare it to 'your' short method. What are the similarities and differences? Unpick the steps and show the children how the short method reduces the steps.

Children should be able to:

- Approximate before they calculate, and make this a regular part of their calculating, going back to the approximation to check the reasonableness of their answer: e.g:

  "**346 x 9** is approximately 350 x 10 = **3500**"
  Record an approximation to check the final answer against it.

- Multiply multiples of ten and one hundred by a single-digit, using their multiplication table knowledge.
- Recall all the times tables **up to 12 x 12**

Key Vocabulary

Groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative law, sets of, lots of, equal groups, times as big as, once, twice, three times…, partition, total, multiple, product, tens, ones, value, hundreds, thousands, inverse, factor

Multiplication

<u>Key Skills needed for multiplying at Year 4:</u>

- Count in multiples of 6, 7, 9, 25 and 1000
- Recognise and recall all multiplication facts up to 12 x 12
- Recognise place value in numbers up to 4 digits
- Use place value, known facts, and derived facts to multiply mentally eg multiply by 1, 10, 100; multiply by 0 or to multiply 3 numbers
- Recognise and use factor pairs in mental multiplication
- Use commutative law, distributive law and associative law to solve mental and written calculations
    - **Commutative:** 3 x 6 = 6 x 3
    - **Distributive:** 39 x 7 = (30 x 7) + (9 x 7)
    - **Associative:** (2 x 3) x 4 = 2 x (3 x 4)
- Combine their knowledge of number facts and the laws of arithmetic to solve calculations such as: 2 x 6 x 5 = 10 x 6 = 60
- Multiply 2 digit and 3 digit numbers by a single digit using formal written methods
- Solve problems involving multiplying and adding which include integer scaling problems and harder correspondence problems such as x objects are connected to y objects
- Solve 2 step multiplication problems in context, choosing the appropriate operation, working with increasingly harder numbers
- Multiply whole numbers by 10, 100 and 1000

Multiplication

Year 5: Multiply up to 4 digits by 1 or 2 digits

Progressing from short to long multiplication

- Children need to be taught to approximate first, e.g. for **72 x 38**, they will use rounding:

  72 x 38 is approximately 70 x 40 = **2800**,

  and use the approximation to check the reasonableness of their answer against.

Short multiplication for multiplying by a single-digit:

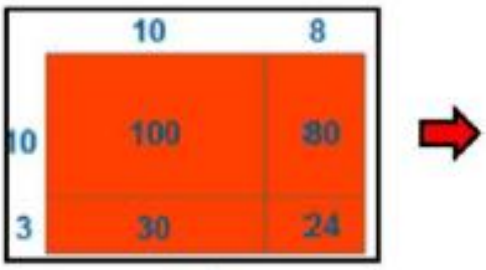
327 x 4 = 1308

(300 x 4) + (20 x 4) + (7 x 4)
1200 + 80 + 28 = 1308

	3	2	7
×			4
1	3	0	8
	1	2	

Pupils could be asked to work out a given calculation using the partitioning method, and then compare it to 'your' column method. What are the similarities and differences? Unpick the steps and show the children how the short method reduces the steps.

Introduce long multiplication for multiplying by 2-digits:



		1	8	
	×	1	3	
		5	4	
	1	8	0	
	2	3	4	

2

18 x 3 on the 1st row

(8 x 3 = 24, 'carrying' the 2 for twenty, then '1' x 3)

18 x 10 on the 2nd row

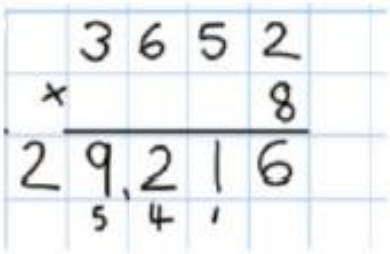
Put a zero in the ones first, then say 8 x 10 = 8 tens, and 10 x 10 = 1 hundred.

Arrays could be used to introduce long multiplication, as the relationship can be seen in the answers in each row.

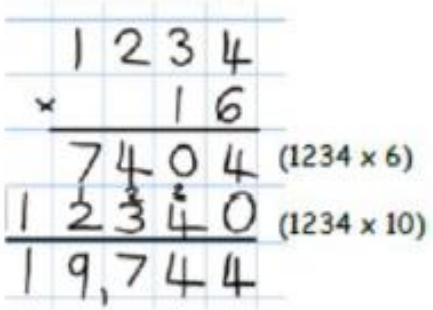
Multiplication

Moving towards more complex numbers

Short multiplication:



Long multiplication:



Key Vocabulary

Groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative law, sets of, lots of, equal groups, times as big as, once, twice, three times..., partition, total, multiple, product, tens, ones, value, hundreds, thousands, inverse, factor, square, cube, integer, decimal, short multiplication, long multiplication, carry, composite numbers, prime numbers, prime factor

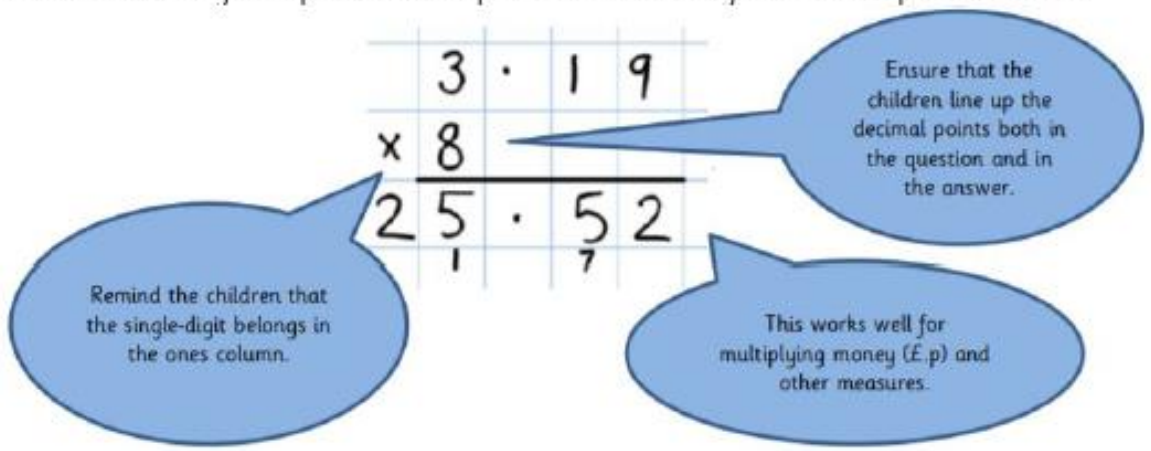
Key skills for multiplication at Year 5:

- Identify multiples and factors, including finding all factor pairs of a number and common factors of two different numbers.
- Recall and use multiplication facts for all the multiplication tables up to 12 x 12.
- Know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.
- Be able to establish whether a number up to 100 is prime and recall prime numbers up to 19.
- Multiply numbers up to 4-digits by a one or two-digit number using a formal written method including long multiplication for 2-digit numbers.
- Multiply numbers mentally drawing upon known facts.
- Use commutative law, distributive law and associative law to solve mental and written calculations:
  Commutative: 3 x 6 = 6 x 3 **Distributive:** 39 x 7 = (30 x 7) + (9 x 7)
  Associative: (2 x 3) x 4 = 2 x (3 x 4)
- Combine their knowledge of number facts and the laws of arithmetic to solve calculations such as: 2 x 6 x 5 = 10 x 6 = 60
- Multiply whole numbers and those involving decimals by 10, 100 and 1000.
- Recognise and use square numbers and cube numbers, along with the notation for these numbers; (^{2}- squared) (^{3}- cubed).
- Solve problems involving multiplication including using their knowledge of factors and multiples and squares and cubes.
- Solve problems involving a combination of operations, choosing and using calculations and methods appropriately.

Multiplication

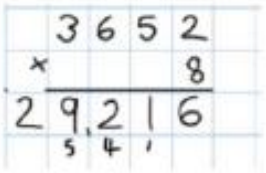
Year 6: Short and long multiplication (as in Y5) and multiply decimals with up to 2 decimal places by a single digit

For short and long multiplication examples with whole integers see: Multiplication Year 5

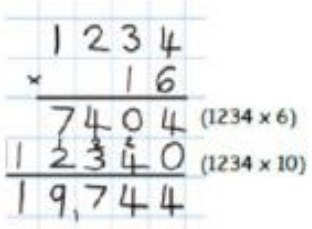


Children will be able to:

- Use rounding and place value to make approximations before calculating and use these to check against answers and their suitability.
- Use **short** multiplication (see Y5):
  1. To multiply numbers with more than 4-digits by a single-digit.
  2. To multiply money and measures.
  3. To multiply decimals with up to 2 decimal places by a single-digit.



- Use **long** multiplication (see Y5):
  1. To multiply numbers with at least 4-digits by a 2-digit number.



Key Vocabulary

Groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative law, sets of, lots of, equal groups, times as big as, once, twice, three times…, partition, total, multiple, product, tens, ones, value, hundreds, thousands, inverse, factor, square, cube, integer, decimal, short multiplication, long multiplication, carry, composite numbers, prime numbers, prime factor

Multiplication

Key Skills for multiplication at Year 6:

- Recall and use multiplication facts for all times tables up to 12 x 12 (as in Y4 & Y5)
- Use all the multiplication facts to calculate mathematical statements in order to maintain fluency of these
- Multiply multi-digit numbers up to 4 digits by a 2 digit number using the formal written method of long multiplication
- Perform mental calculations with mixed operations and large numbers which are increasingly complex
- Explore the order of operations using brackets and understand the different effects these can have to an answer
- Identify common factors, common multiples and prime numbers
- Use their knowledge of the order of operations to carry out the four operations with confidence
- Solve multi step problems in a range of contexts choosing appropriate combinations of operations and methods
- Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy
- Round any integer to a required degree of accuracy

Division

Year 1: Group and Share small quantities

Children are given the opportunity to use concrete objects, diagrams and pictorial representations to solve problems involving both **grouping** and **sharing** linked to early division.

÷

Grouping:

How many groups of 4 can be made with 12 stars?

3 groups of 4 can be made with 12 stars

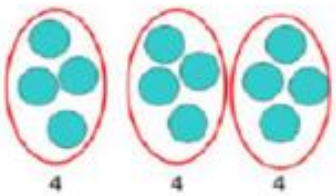
An example division problem in a familiar context:

There are 6 children on this table and there are 18 pieces of fruit to share between them. If we share the pieces of fruit equally, how many pieces will each child get?

Can the children work out and given a division statement ...?
"18 shared between 6 children gives you 3 each."

Sharing:

What is 12 shared between 3?



12 shared between 3 is 4

Children should:

- Use lots of practical apparatus, arrays and picture representations for early division.
- Be taught the difference between '**grouping**' objects (How many groups of 2 can you make?) and '**sharing**' (share these sweets between 2 people).
- Be taught to count in 2s, 5s and 10s.
- Find **half** of a group of objects by sharing into 2 equal groups.

Key Vocabulary

Share, share equally, one each, two each..., group, groups of, lots of, array, half

Key Skills for division at Year 1:

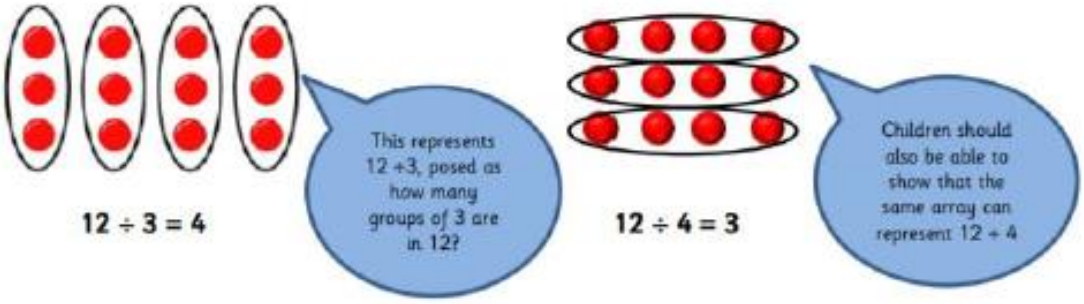
- Solve one step division problems, calculating the answer by using concrete objects, pictorial representations and arrays with the support of the teacher
- Through grouping and sharing small quantities, children begin to understand division and finding simple fractions of objects, numbers and quantities
- Children can make connections between arrays, number patterns and counting in 2s, 5s and 10s.

Division

Year 2: Grouping and sharing using the ÷ and = sign

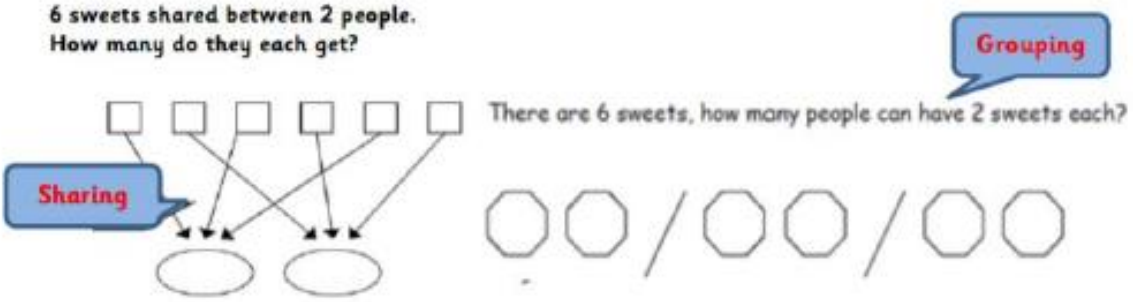
Children will be given opportunity to use objects, arrays, diagrams and pictorial representations and grouping on a number line.

Arrays:



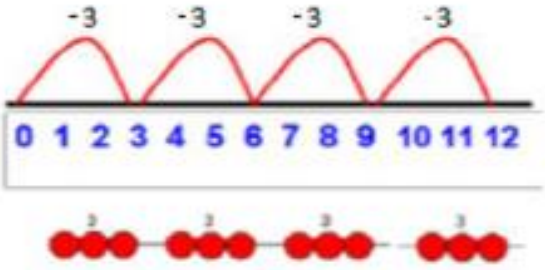
Know and understand sharing and grouping:

6 sweets shared between 2 people.
How many do they each get?



Grouping using a number line: (Repeated Subtrcation)

Group from zero in equal jumps of the divisor to find out 'how many groups of Δ in Δ?'. Children could be using bead strings or practical apparatus to solve problems such as:
A CD costs £3. How many CDs can I buy with £12?'
This is an important method to develop understanding of division as grouping.



Pose **12 ÷ 3** as **'How many groups of 3 are in 12?'**

12 ÷ 3 = 4

Key Vocabulary

Share, share equally, one each, two each…, group, groups of, lots of, array, half, divide, divided by, divided into, division, grouping, number line, left, left over

Division

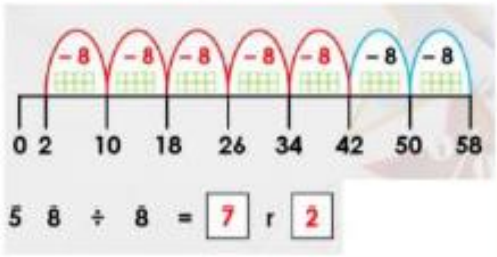
Key Skills for division at Year 2:

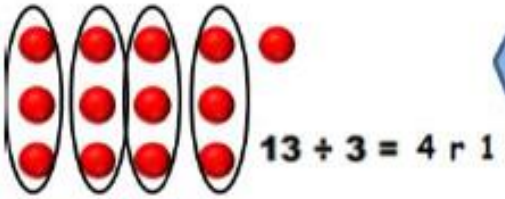
- Count in steps of 2, 3, 5 and 10 from zero and ten from any number
- Recall and use division facts from the 2, 5 and 10 times tables, including recognising odds and evens
- Show that multiplication can be done in any order (commutative) but division is not
- Calculate division statements within the multiplication tables and write them using the division and equals symbols
- Solve division problems using materials, arrays, repeated subtraction, mental methods and multiplication and division facts including problems in context
- Use a variety of language to discuss and describe division.

Division

Year 3: Divide 2 digits by a single digit

Introducing remainders in division:

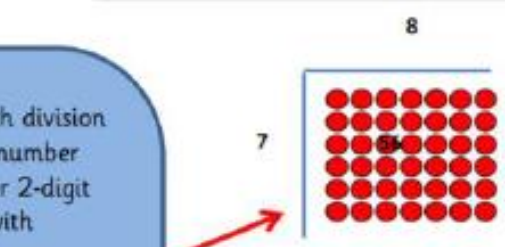




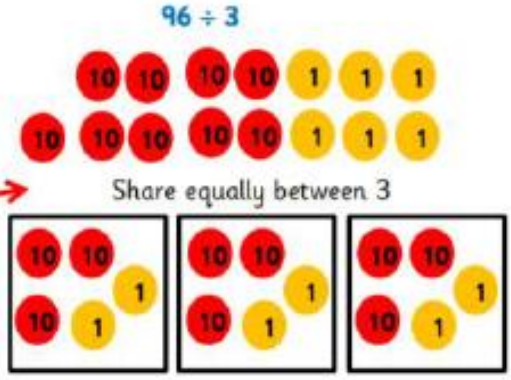
Step 1: Children can continue to work out unknown division facts by grouping on a number line (repeated subtraction) from zero. Children are also now taught the concept of **remainders**, as in the example shown. This should be introduced practically and with arrays, as well as being translated to the number line. Children should work towards calculating some basic division facts with remainders **mentally for the 2s, 3s, 4s, 5, 8s and 10s,** ready for **'carrying' remainders** across within the short division method.

Introducing Short division:

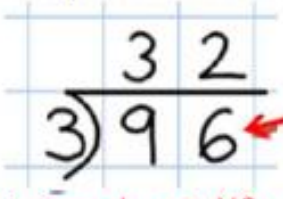
Step 2: Once the children are secure with division as grouping and demonstrate this using number lines, arrays etc.., short division for larger 2-digit numbers should be introduced, initially with carefully selected examples requiring no calculating of remainders. Start by introducing the layout of short division by comparing it to an array.



Short division may also be clearly demonstrated using the equal-sharing structure and place value counters or Dienes apparatus:



Progress from practical application to the formal written method:



Remind children of correct place value, that 96 is equal to 90 and 6, and in short division pose:

- How many 3's in 9? = 3, and record 3 above the 9 tens.
- How many 3's in 6? = 2, and record 2 above the 6 ones.

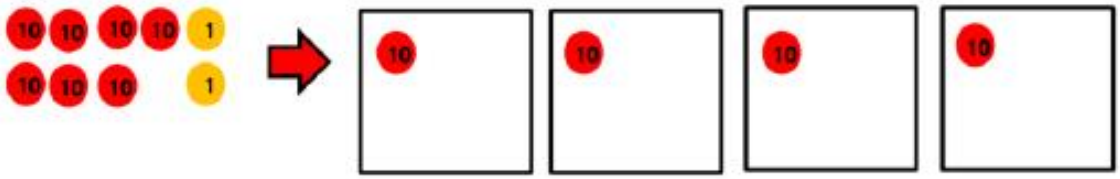
Limit numbers to NO remainders in the final answer OR carried (each digit must be a multiple of the divisor).

Division

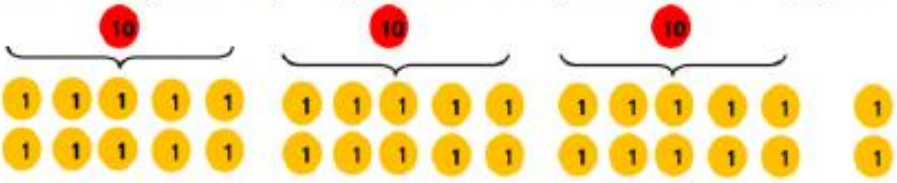
Step 3: Once the children demonstrate a full understanding of remainders, and also the short division method taught, they can then progress to working with the short division method when remainders occur within the calculation (e.g. 96 ÷ 4). The children will need to be taught to 'carry' the remainder onto the next digit. **If needed, children should use the number line to calculate individual division facts that occur which they are not yet able to recall mentally.**

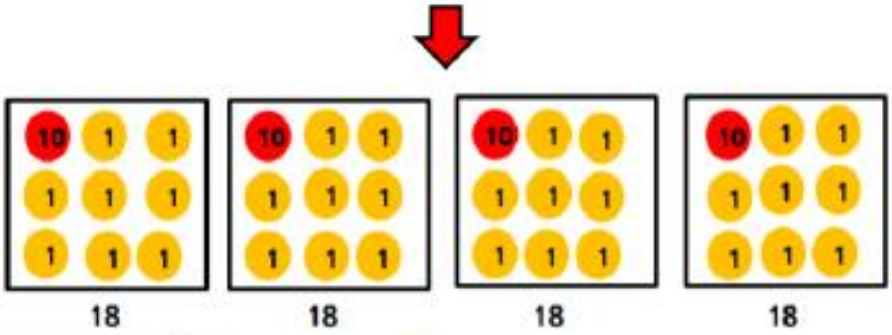
Short division including remainders within the calculation:

72 ÷ 4



1 ten has been given to each of the 4 groups; the 3 remaining tens need to be exchanged for ones:





Real life contexts:

These need to be used routinely to help children gain a full understanding and ability to recognise the place of division and how to apply it to solving problems.

$$4\overline{)72}$$

1 8

Children must be taught to 'carry' the remainder of tens into the ones place value column.

Step 3: Only taught when children can calculate **'remainders'**.

Limit numbers to NO remainders in final answer but with remainders occurring within the number and being carried across.

Division

÷

Key Vocabulary

Share, share equally, one each, two each…, group, groups of, lots of, array, half, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, carry, remainder, multiple

Key Skills for division at Year 3:

- Count in multiples of 2, 3, 4, 5, 8 and 10 from zero
- Recall and use multiplication and division facts from the 2, 3, 4, 5, 8 and 10 times tables and divide multiples of 10 by 10
- Write and calculate number statements for division using the multiplication tables already learnt including 2 digits divided by single digits, drawing on mental methods and progressing to the formal short division method.
- Solve division problems including missing number problems
- To understand and begin to use remainders in division
- Develop mental strategies for calculating divisions; use known times table facts to derive related division facts – inverse.
- Divide whole numbers by 10 or 100
- Solve simple problems in context, deciding which operations and methods to use
- Develop efficient mental methods to solve a range of problems

Division

Year 4: Divide up to 3 digit numbers by a single digit

Initially without remainders

÷

Continue to develop short division

Children must be taught to 'carry' the remainder of tens into the ones place value column.

$$4\overline{)7^{3}2} = 18$$

Short division should only be taught once children have secured the skill of calculating **'remainders'**.

Step 1: Children must be secure with the process of short division for dividing 2-digits by a single digit. **(Those that do not result in a final remainder – see Year 3)** However, the children must understand how to calculate remainders, using this to 'carry' remainders within the calculation process.

Step 2: Children move onto dividing numbers with 3-digits by a single digit, **however problems and calculations provided should not result in a final answer with a remainder at this stage**. Children who exceed this expectation may progress to the Year 5 level.

$$4\overline{)8\,7^{3}2} = 218$$

$$5\overline{)1\,8^{3}5} = 037$$

When the answer for the **first column** is zero (1 ÷ 5, as in the example), children could initially write a zero above to acknowledge its place, and **must always 'carry' the number (1) over to the next digit as a remainder.**

Real life contexts:

These need to be used routinely to help children gain a full understanding and ability to recognise the place of division and how to apply it to solving problems.

Include money and measure contexts when the children are confident.

Division

Key Vocabulary

Share, share equally, one each, two each…, group, groups of, lots of, array, half, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, carry, remainder, multiple, divisible by, factor

Key Skills for division at Year 4:

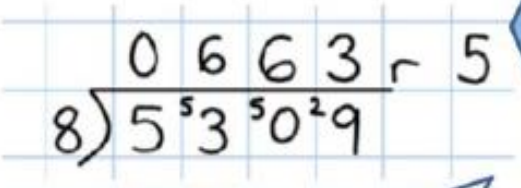
- Count in multiples of 6, 7, 9, 25 and 1000.
- Recall and use division facts for **all the multiplication tables up to 12 x 12**.
- Recognise place value of digits in up to 4-digit numbers.
- Use place value, known facts and derived facts to divide mentally, e.g. divide by 1, 10 and 100.
- Recognise and use factor pairs in mental calculations, understanding how they link to division.
- Practise mental methods and extend known facts to 3-digit numbers to derive facts: for example 600 ÷ 3 = 200 can be derived from 2 x 3 = 6
- Divide 2-digit and 3-digit numbers by single-digit numbers using formal written layouts.
- Solve 2-step division problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as three cakes shared equally between 10 children.
- Divide whole numbers by 10, 100 and 1000.

Division

Year 5: Divide up to 4 digit numbers by a single digit

Including those with remainders.

Short division, including remainder answers:



The answer to 5309 ÷ 8 could be expressed as:

1. **663 and 5/8,**
2. **663 r 5,**
3. **663.625 (decimal)**
4. **Or rounded as appropriate to the problem involved.**

Understand 663 is the **quotient** and the 5/8, r 5 or .625 is the remainder.

Short Division Step 3

Short division with remainders: Now that children are introduced to examples that give rise to remainder answers, division needs to have a real life problem solving context, where pupils consider the meaning of the remainder and how to express it. For example: as a fraction, a decimal, or as a rounded number or value, depending upon the context of the problem.

See Year 6 and end of video clip for how to continue the short division to give a decimal answer for children who are confident.

Real life contexts:

These need to be used routinely to help children gain a full understanding and ability to recognise the place of division and how to apply it to solving problems.

Include money and measure contexts when the children are confident.

If children are confident and accurate:

- Introduce long division for children who are ready to divide any number by a 2-digit number (e.g. 2678 ÷ 19). This is a Year 6 expectation – see Y6.

Division

Key Vocabulary

Share, share equally, one each, two each…, group, groups of, lots of, array, half, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, carry, remainder, multiple, divisible by, factor, quotient, integer, prime number, prime factors, composite number (non-prime)

Key Skills for division at Year 5:

- Identify multiples and factors, including finding all factor pairs of a number and common factors of two different numbers; understanding their link to division.
- Recall and use multiplication facts for all the multiplication tables up to 12 x 12.
- Know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.
- Be able to establish whether a number up to 100 is prime and recall prime numbers up to 19 – know that these are only divisible by 1 and themselves.
- Divide numbers up to 4-digits by a one or two-digit number using a formal written method
- Divide numbers mentally drawing upon known facts.
- Combine their knowledge of number facts and the laws of arithmetic to solve calculations such as: 60 ÷ 6 ÷ 5= 10 ÷ 5 = 2
- Divide whole numbers and those involving decimals by 10, 100 and 1000.
- Use multiplication and division as inverses.
- Interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions (quotients), as decimals or by rounding.
- Solve problems involving a combination of operations, choosing and using calculations and methods appropriately.

Division

Year 6: Divide at least 4 digit numbers by 1 and 2 digit numbers

Including decimal numbers and quantities

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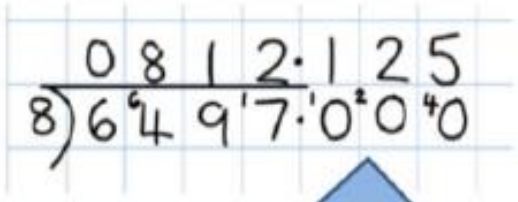
Short division for dividing by a single digit;

6497 ÷ 8

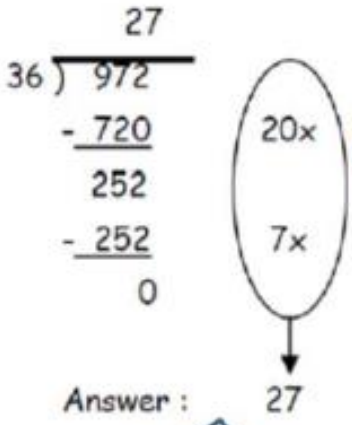


Calculating a decimal remainder:
In this example, rather than expressing the remainder as **r 1**, a decimal point is added after the ones because there is still a remainder, and the one remainder is carried onto zeros after the decimal point (to show there was no decimal value in the original number). Children keep dividing to an appropriate degree of accuracy for the problem being solved.

Short Division Step 4

Short division with remainders: children should continue to use this method, but with numbers to at least 4-digits, and understand how to express remainders as quotients, decimals, whole number remainders or rounded numbers relevant to given problems. Real life problem solving contexts need to be the starting point, where pupils have to consider the most appropriate way to express the remainder.

Introducing long division by chunking for dividing by 2 digits;



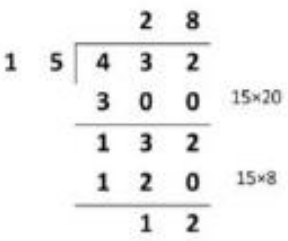
Digits must be aligned in place value for subtracting.

- Find out 'How many 36's are in 972?' by subtracting 'chunks' of 36, until zero is reached (or until there is a remainder).
- Teach children to write a 'useful list' first at the side that will help them decide what 'chunks' to use, e.g:

'Useful' list: 1 x 36 = 36
10 x 36 = 360
100 x 36 = 3600

- Introduce the method in a simple way by limiting the choice of chunks to 'Can we use 10 lots? Can use 100 lots? As children become confident with the process, encourage more efficient chunks to get to answer more quickly (e.g. 20x, 5x), and expand on their 'useful' lists.

432 ÷ 15 becomes



$\frac{12}{15} = \frac{4}{5}$

Answer: $28\frac{4}{5}$

Digits must be aligned in place value for subtracting.

Division

Further examples of long division

432 ÷ 15 becomes

```
      2  8  r 12
   ┌──────────
1 5 │ 4  3  2
      3  0  0
   ──────────
      1  3  2
      1  2  0
   ──────────
         1  2
```

Answer: 28 remainder 12

Where **remainders** occur, pupils should express them as fractions, decimals or use rounding, depending upon the problem.

432 ÷ 15 becomes

```
      2  8 · 8
   ┌──────────
1 5 │ 4  3  2 · 0
      3  0  ↓
   ──────────
      1  3  2
      1  2  0  ↓
   ──────────
         1  2  0
         1  2  0
   ──────────
               0
```

Answer: 28·8

Key Vocabulary

Share, share equally, one each, two each…, group, groups of, lots of, array, half, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, carry, remainder, multiple, divisible by, factor, quotient, integer, prime number, prime factors, composite number (non-prime), common factor

Key skills for dividing at Year 6:

- Recall and use division facts for all the multiplication tables up to 12 x 12 (as Y4 and Y5).
- Use all the multiplication facts to calculate mathematical statements based on the inverse of division in order to maintain fluency at these.
- Divide at least 4-digits by a single digit and 2-digit number using the formal written methods of short and long division.
- Interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions (quotients), as decimals or by rounding.
- Perform mental calculations with mixed operations and large numbers, which are increasingly complex.
- Explore the order of operations using brackets and understand the different effects these can have to an answer.
- Identify common factors, common multiples and prime numbers, relating common factors to equivalent fractions for remainders in division.
- Use their knowledge of the order of operations to carry out calculations involving the four operations with confidence.
- Solve multi-step problems in a range of contexts, choosing appropriate combinations of operations and methods.
- Use estimation techniques to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.
- Round any integer to a required degree of accuracy.
- Use written division methods in cases where the answer has up to 2 decimal places.